Testing the method of measuring inclination angles of disk galaxies

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Abstract

Determining the accurate inclination angle of disk galaxies is important when projection effects need to be considered and when correcting for internal dust extinction. In observations, inclination angles are determined by assuming that the disk is circular when viewed face on and has a finite intrinsic thickness. These assumptions are often inaccurate, and measurements of apparent galaxy disk shapes are often biased by non-axisymmetric features and faint structures in the outskirt that do not belong to the disk. Using synthetic images of disk galaxies in the Illustris TNG50 cosmological simulation, we apply the same method used in observations to estimate inclination angles and compare the results with the intrinsic inclination angles derived from the simulation, in order to evaluate the robustness of the method. We find that the assumption regarding disk thickness has a small effect on the accuracy of inclination angle measurements except at high inclination angles. At intermediate inclination angles, the intrinsic inclination angles are consistent with the observed inclination angles. At high inclination angles, the intrinsic inclination angles are systematically larger than the observed ones due to the presence of bulges and/or other kinematically hot structures. At low inclination angles, the observed inclination angles are systematically higher than the intrinsic inclination angles due (at least in part) to disks not being circular when viewed face on.

1. Introduction

Most of the disk galaxies observed are not viewed face on. This makes measuring accurate inclination angles important for studies where projection effects should be accounted for and useful when correcting for internal dust extinction. Galaxy scaling relations are crucial in understanding galaxy formation and evolution, but the scaling relations that relate size, luminosity, surface brightness, and bulge-to-disk ratio have biases introduced due to dust and
inclination (Pastrav et al., 2013a,b; van der Giessen et al., 2022; Pastrav, 2020). Additionally, measuring the parameters of bars and spiral arms requires correcting for projection effects which relies on the inclination angle (Gadotti et al., 2007).

In observations, the inclination angle of disk galaxies is often calculated by assuming that the disk is circular (when viewed face on) and has a finite thickness. The inclination angle \( i \) can then be derived from Hubble’s classical equation (Hubble, 1926)

\[
\cos^2 i = \frac{q^2 - q_0^2}{1 - q_0^2},
\]

where \( q \) is the observed axis ratio in galaxy images, which is defined as the minor axis over the major axis, and \( q_0 \) is the intrinsic axis ratio defined as the observed \( q \) when viewed edge-on (Fouque et al., 1990). The apparent ellipticity of the disk is related to \( q \) by the equation \( e = 1 - q \).

However, the assumption that disks are circular when they are viewed face on is often violated. Rix and Zaritsky (1995) constructed a sample of face-on galaxies by comparing the galaxy’s observed H1 width and its characteristic velocity. They found that most disks feature non-axisymmetric structures and have a mean ellipticity of \( e = 0.045 \). In galaxies such as NGC 1300 (left panel of Figure 1), it is difficult to envision an axisymmetric disk that averages the strong spiral arms and bars. These strong non-axisymmetric features can also hamper accurate measurements of the disk ellipticity due to the difficulties in automatically choosing a proper radial regime over which the average ellipticities are computed. The common practice is to use the outermost isophotes for such a purpose. However, other morphological components and features such as thick disks, stellar halos, and imprints of galaxy interactions may come into play if the images are deep enough. For instance, the disk of the galaxy NGC 4596 (right panel of Figure 1) is embedded in a prominent spheroidal component, which leads to underestimates of the ellipticity of the disk if one chooses to compute it over the outermost isophotes.

Moreover, some studies assumed a constant disk thickness \( q_0 \) for the sake of simplicity when measuring inclination angles. Ho et al. (2011) use the classical equation (Equation 1) to determine the inclination of galaxies. For simplicity, they use a value of \( q_0 = 0.2 \), similar to Noordermeer and van der Hulst (2007). However, the thickness of a disk is not constant across different galaxy types, so this assumption has the potential to introduce bias into their sample of galaxies. The disk thickness is found to be a function of morphological type (Heidmann et al., 1972; Bottinelli et al., 1983; Yuan and Zhu, 2004). Sánchez-Janssen et al. (2010) find that low-mass galaxies are systematically thicker below \( M_* \approx 2 \times 10^9 \) M\(_\odot\). Rodríguez and Padilla (2013) find that the thickness of disks increases with luminosity and size. These findings indicate that measurements of the inclination angles of galaxies may be inaccurate when assuming that disks have a universal \( q_0 \).

Other factors like stellar population effects in optical images or uneven dust extinction on the near and far half of the galaxies should also play a role in biasing the ellipticity measurements. Redder spiral galaxies are found to have higher dust extinction, which may
further bias ellipticity measurements (Rodríguez and Padilla, 2013).

The aforementioned issues put the measurements of inclination angles into question. Although we do not expect that the measurements are generally inaccurate, it needs to be clarified what the typical magnitude of uncertainties is and under what conditions the assumptions induce significant errors.

In observations, when working with images where we only have projected two-dimensional flux distribution, there are no direct methods to validate the measurements. Alternatively, in simulations, we can make full use of the three-dimensional information of the galaxies. We can then compare the “ground truth” obtained from the simulated galaxies with the ones measured with mock data to answer the above questions.

We make use of the IllustrisTNG (TNG) simulations (Nelson et al., 2019b), which are cosmological hydrodynamical simulations that evolve a mock universe from shortly after the Big Bang until the present. The smallest of these simulations is the TNG50 simulation, which uses a cubic volume with a roughly 50 Mpc side length and provides the greatest detail on the structural properties of galaxies. The galaxy population produced by the TNG50 simulation is qualitatively consistent with observations (Pillepich et al., 2019; Nelson et al., 2019a). Rodriguez-Gomez et al. (2019) find that the IllustrisTNG simulation reproduces the optical morphologies of galaxies such that the median trends of the morphological, size and shape parameters with stellar mass are within the $\sim 1\sigma$ scatter of the observed trends.

The HSC Project 405 has generated synthetic HSC-joint images of galaxies from the TNG50 simulation, using the SKIRT9 Monte Carlo radiative transfer code (Camps and Baes, 2020).
These galaxies are injected into real HSC cutouts so that these mock images have consistent properties with observed HSC images (Aihara et al., 2018, 2022). The mock images of the TNG50 galaxies consist of stellar masses $\log(M_*/M_\odot) > 9$. For this paper, we use the images generated at snapshot 91, which corresponds to a redshift of $z = 0.1$. Each galaxy has mock images produced from four different viewing angles.

In this work, we test the method of measuring inclination angles of disk galaxies using these mock images generated from the TNG50 simulation for four viewing angles relative to the simulation box. We calculate the inclination angles of each galaxy from synthetic images using the same methods used for observations. We compare these inclination angles with the intrinsic inclination angles of the galaxies derived from the simulation. We explore the differences between the observed inclination angles and the intrinsic ones as a function of intrinsic galaxy properties, the optical bands at which they are measured, and the inclination angles themselves. The paper is organized as follows: In Section 2 we describe our sample of galaxies and in Section 3 we describe the methods we use to determine the inclination angles of the galaxies. In Section 4 we present our results and offer conclusions in Section 5.

2. Sample selection

We start by selecting disk galaxies from the TNG50 simulation. The parameter $\kappa_{\text{rot}}$ quantifies the kinematic morphology of a galaxy by determining the fraction of the total kinetic energy contributed by the azimuthal component of the stellar velocities:

$$\kappa_{\text{rot}} = \frac{1}{K} \sum_i \frac{1}{2} m_i \left( \frac{j_{z,i}}{R_i} \right)^2,$$

where $K$ represents the stellar component’s total kinetic energy, $m_i$ is the mass of a particle, $j_{z,i}$ represents the $z$-component of the specific angular momentum, and $R_i$ is the projected radius (Sales et al., 2010; Rodriguez-Gomez et al., 2017). We select a sample of disk galaxies with $\kappa_{\text{rot}} > 0.5$ within 10 times of stellar half mass radii following Zhao et al. (2020). This results in a sample of 2207 disk galaxies from the 3047 TNG50 galaxies. As each galaxy has mock images produced from four different viewing angles, we can calculate the inclination angle for each of the four images separately. This results in a sample of 8828 disk galaxies.

3. Methods

3.1. Inclination Angle of Galaxies from the Mock Images

We apply an automated non-parametric light profile extraction pipeline called *AutoProf* to our sample of disk galaxies (Stone et al., 2021). While the main purpose of the package is to extract reliable light profiles of galaxies from images, we only make use of its functionality to obtain the global ellipticity of the galaxy. The global position angle and ellipticity are obtained during its “initialization” stage, where the code obtains an initial guess of geometric
parameters for subsequent fitting of isophotes across the galaxy. We do not repeat the full
details (see their Section 2.4) but only provide a brief description here. The initialization
determines an average ellipse that traces the flux distribution of the outermost part of
the galaxy. Specifically, it is a two-step process. The code first determines the position
angle by finding the average of the phase angles of $\mathcal{F}_i$ of flux along a circular path in the
galaxy outskirts, where $\mathcal{F}_i$ is the $i^{th}$ fast-Fourier transform coefficient of flux values along the
isophotal path. Then, with the position angle being fixed, the ellipticity is obtained as the
one that minimizes $\mathcal{F}_2/\mathcal{F}_0$. The upper-left panel of Figure 2 shows how the position angle
is calculated and the lower-left panel of Figure 2 shows how the ellipticity is selected as the
minimum of $\mathcal{F}_2/\mathcal{F}_0$. The initial ellipse fit to the galaxy is shown on the right side of Figure 2.
The uncertainties of the position angle and ellipticity are estimated by performing the same
procedure above but perturbing the semi-major axis length of the best-fit ellipse within one
FWHM of the PSF.

![Figure 2: AutoProf outputs from the initialization stage. The left side shows how the position angle is calculated as the average of the phase angles in the galaxy outskirts and how the ellipticity is obtained by minimizing $\mathcal{F}_2/\mathcal{F}_0$. The right side shows the initial ellipse fit to the galaxy.](image)

The data products of HSC405 provide a PSF model and a mask image for each image.
Therefore, we do not allow AutoProf to build the PSF model on its own, and set use the
FWHM of the PSF models we obtain. The FWHM is calculated by fitting a two-dimensional
Gaussian to the PSF models. We also provide the mask image as input to AutoProf. At high
values of $q$ or low inclination angles, the standard AutoProf pipeline preferentially selects
certain ellipticities, probably due to inadequate numerical solutions of the optimization. We
therefore made adjustments to the AutoProf code to better determine the ellipticities and
refer the reader to Appendix A for an explanation of how the pipeline was modified to
remove such biases.
AutoProf provides four data quality checks on the FFT coefficients, the light symmetry, the initial fit compare, and the isophote variability. All galaxies with four fails are removed from the sample. We determine whether the galaxies with two or three fails have good fits by eye, and remove the cases with bad fits. The bad fits removed are due to interacting galaxies or bright foreground stars. A sample of galaxies with one fail is examined by eye. The ellipticities of the galaxies in this sample have good fits, and are therefore left in our sample.

We determine $q_0$ as the lower limit of $q$ distributions of galaxies at given stellar mass bins. Therefore, our $q_0$ is a function of stellar mass. Due to the small sample size, we are not able to further explore the dependence of $q_0$ on other galaxy properties such as color in each stellar mass. We do not use the values adopted in the observational studies since it is not yet clear that the thickness of TNG galaxy disks are consistent with the observed ones. These values of $e$ and $q_0$ allow us to calculate the inclination angle of each galaxy in a method consistent with the common practice in observations using Equation 1. We split the sample into six mass bins of equal size and set $q_0$ to be the first percentile of $q$ in each mass bin. Any galaxy with $q < q_0$ is removed from the sample. This results in a sample of 8545 galaxies.

3.2. Inclination Angle of Galaxies from the Simulations

The intrinsic inclination angle is defined as the angle between the line of sight and the disk norm of the galaxies. We use the stellar angular momentum vector as a proxy of the disk norm. The stellar angular momentum of the simulated galaxies are calculated inside 10 times of the half stellar mass radii.

3.3. The intrinsic axis ratios

We calculate the intrinsic axis length $A > B > C$ of the galaxies. First, we find the principal axis based on the inertia tensor of the stellar mass particles of galaxies. Then, we calculate the second order moment of the mass distribution along the three principal axes. We compute the face on axis ratio as $B/A$. Additionally, we calculate the intrinsic thickness, $q_0 = \frac{C}{\sqrt{A+B}}$.

4. Results and Discussion

4.1. Disk thickness $q_0$

In Figure 3, we show the observed axis ratio $q$ as a function of stellar mass using Y-band images. We follow the methods in Section 3.1 to measure $q_0$ for galaxies with different stellar masses. The values of $q_0$ for each mass bin are marked by the red stars. We find that $q_0$ is not a constant and shows a ‘U-shape’ dependence on the stellar mass of galaxies. This is consistent with the ‘U-shape’ distribution found in Sánchez-Janssen et al. (2010). Galaxies with lower stellar masses ($\sim 10^9 M_\odot$) tend to be more spheroidal. Additionally, galaxies
with larger stellar masses have a higher apparent axial ratio due to the larger contribution of spheroidal components (Sánchez-Janssen et al., 2010). The five values of $q_0$ measured as a lower bound of $q$ at a given mass are $q_0 = 0.26$ in the $9 - 9.2 \log M_\odot$ stellar mass range, $q_0 = 0.22$ in the $9.2 - 9.5 \log M_\odot$ stellar mass range, $q_0 = 0.17$ in the $9.5 - 9.8 \log M_\odot$ stellar mass range, $q_0 = 0.15$ in the $9.8 - 10.2 \log M_\odot$ stellar mass range, $q_0 = 0.14$ in the $10.2 - 11 \log M_\odot$ stellar mass range, and $q_0 = 0.20$ for stellar masses greater than $11 \log M_\odot$.

Figure 3: The observed axis ratio $q$ as a function of stellar mass where the blue lines are the kernel density estimate. The red stars show the values of $q_0$ measured for each mass bin. The dashed red lines show how the mass bins are divided.

In Figure 4, we show how our values of $q_0$ affect the inclination angle as compared with the inclination angle assuming a constant value of $q_0 = 0.2$. The top panel compares the inclination angle $i$ calculated when $q_0 = 0.2$ and the $q_0$ value obtained for our sample as a function of stellar mass, and the bottom panel shows the residual as a function of $i$. As our measured $q_0$ values do not significantly deviate from 0.2, the differences are mostly negligible at low to intermediate inclination angles. The effect of the value of $q_0$ on the inclination angle $i$ becomes most pronounced at high inclination angles. The difference in inclination angles at our highest and lowest values of $q_0$ is $\sim 5^\circ$ at the highest inclination angles. So, the practice of adopting a constant $q_0$ is actually a reasonable choice, as long as the adopted $q_0$ does not deviate from the true $q_0$ significantly.

We examine the difference between inclination angles when using the $q_0$ we measure as
Figure 4: Comparison of inclination angles with a constant $q_0 = 0.2$ and $q_0$ measured as a function of stellar mass. The top panel compares the inclination angle calculated $i$ when $q_0 = 0.2$ versus the $q_0$ measured for our sample as a function of stellar mass. The five colors represent the five $q_0$ values we measure. The black line shows a 1:1 line to help guide the eye. The bottom panel shows the difference between the inclination angle $i$ when $q_0 = 0.2$ and $q_0$ measured for our sample as a function of stellar mass as a function of $i$. The black line shows $\Delta i = 0$. 
a function of stellar mass and the intrinsic $q_0$. In Figure 5, we show their differences as
a function of intrinsic inclination angles. The difference in inclination angles is higher at
higher intrinsic inclination angles. The mean difference in $i$ is 1.43 degrees and the standard
deviation is 2.46 degrees. The inclination angle calculated using the intrinsic $q_0$ is system-
atically larger than the intrinsic $q_0$ we measure as a function of stellar mass. The values
of $q_0$ we measure as a function of stellar mass generally result in accurate inclination angle
measurements except for highly inclined galaxies. These results show trends consistent with
Figure 4.

![Figure 5: The difference between inclination angles when using the $q_0$ we measured as a function of stellar mass and the intrinsic $q_0$ as a function of intrinsic inclination angle.]

### 4.2. Comparison of inclination angles

We show the distributions of cosine of the inclination angle calculated in a way similar to
observations and cosine of the intrinsic inclination angle in Figure 6. Because the galaxies are
viewed at random angles, $\cos i$ should have a uniform distribution. However, the observed
$\cos (i_{\text{observed}})$ does not have a uniform distribution, unlike the intrinsic ones. The reason for
this is discussed below.

In the top panel of Figure 7, we compare the intrinsic inclination angle and the inclination
angle calculated in a way similar to observations. In the bottom panel of Figure 7, we show
the difference in inclination angle $\Delta i$ between the observed inclination angle and the intrinsic inclination angle as a function of the intrinsic inclination angle. We show the results obtained from Y-band images here, as the Y-band should have the lowest levels of dust extinction.

At low inclination angles, the observed inclination angle is systematically larger than the intrinsic inclination angle. Additionally, $\Delta i$ decreases as the intrinsic inclination angle increases. The origin of the systematic bias will be discussed further in Section 4.3. At high inclination angles, the intrinsic inclination angle is larger than the observed inclination angle. The difference between the two inclination angles increases as the intrinsic inclination angle increases. In Section 4.4, we will demonstrate that the main cause of this difference is the presence of bulges. These two systematic biases at the low and high inclination angle end result in the non-uniform distribution of $\cos (i_{\text{observed}})$ seen in Figure 6.

The difference in inclination angle $\Delta i$ has an upper and lower bound (see the dotted blue line in the bottom panel of Figure 7) because the intrinsic and observed inclination angle must be between 0 and 90 degrees. Recall that we determined $q_0$ as the first percentile so that there were $\sim 1\%$ galaxies with $q < q_0$. We simply remove these galaxies from the sample in subsequent calculations of the inclination angles. So, galaxies that fall into the forbidden region at the high inclination end were simply clipped out. We are not totally sure about the behaviour of galaxies hitting the boundary ($q \rightarrow 1$) at the low inclination end, because it is solely determined by AutoProf. However, we do not observe overdensity of galaxies at the boundary of the forbidden zone at this end, hinting that AutoProf is at least not biased at the boundary. Also, the fact that both the mean bias and lower $1\sigma$ significantly deviate from zero suggests that this effect cannot explain the offset at the low inclination

![Figure 6: The distribution of cosine of the intrinsic inclination angle (left) and the distribution of cosine of the observed inclination angle (right).](image)
angle end. This is also true for the high inclination angle end, because not only is the mean
skewed toward negative values but also the mode is skewed toward the same direction and
this cannot be accounted for by the clipping of galaxies in the forbidden zone.

In Figure 7, we show the running mean of $\Delta i$ as a function of the intrinsic inclination angle
from the G, I, and Y band images. We find that the G and I bands follow the same trend
as the Y band. This indicates that effects related to wavelength, including both stellar
populations and dust extinction, do not play a noticeable role in biasing inclination angle
measurements. Further investigations require deriving the dust content of the galaxies.

4.3. Low inclination angles

Figure 7 shows that the observed inclination angle is higher than the intrinsic inclination
angle at low inclination angles. There is a chance that this may partially be due to a
numerical issue in AutoProf. Using the default setting of AutoProf, we find that the
ellipticities of nearly face-on galaxies are clustered around some fixed values, although we
modified the codes and key parameters to remedy this clustering. On the other hand, it may
also be due to the assumption that the disk is circular when viewed face on in Equation 1.
This equation assumes that $q = 1$ when the galaxy is face on ($i = 0$). If disk galaxies are
not intrinsically circular when viewed face on, $q$ is smaller than one. Thus, face-on galaxies
or near face-on galaxies would have a higher observed inclination angle.

Two examples of these galaxies can be seen in Figure 8. The galaxy on the left has an
intrinsic inclination angle of 10° and an observed inclination angle of 60°. The galaxy on the
right has an intrinsic inclination angle of 3° and an observed inclination angle of 45°. Ad-
ditionally, both of these galaxies have strong non-axisymmetric features, which may further
bias measurement of the galaxy ellipticity.

We show how the intrinsic shape of galaxies correlates with the difference between the
observed inclination angle and the intrinsic inclination angle $\Delta i$ in Figure 9. We use $1 - B/A$
(see Section 3.3 for the definition) to quantify the deviation of a galaxy from a circular shape.
At low inclination angles, $\Delta i$ is larger for galaxies that are less circular ($1 - B/A$ is higher).
This indicates that at the low inclination angle end, the difference in inclination angles is
due (at least in part) to the assumption that disk galaxies are circular when viewed face on.

4.4. High inclination angles

As seen in Figure 7, at higher inclination angles, there is a larger difference between the
observed inclination angle and the intrinsic inclination angle such that the intrinsic inclina-
tion angle is larger than the observed inclination angle. The decrease in mean $\Delta i$ as the
intrinsic inclination angle increases may be due to the presence of bulges or other round
(kinematically hot) structures in the galaxies. When a bulge is face on it will not affect
the measurements of ellipticity, but as the inclination angle increases the presence of bulges
will lead to underestimates of the disk ellipticity of the galaxy. This results in the observed
Figure 7: Comparison of the observed inclination angle and the intrinsic inclination angle. Top panel: The intrinsic inclination angle versus the observed inclination angle using a logarithmic scale with contours overlaid in black. The red dashed line shows a 1:1 line. Bottom panel: The difference in inclination angle $\Delta i$ between the angle calculated in a way similar to observation and the intrinsic inclination angle as a function of the intrinsic inclination angle. The dotted, red line shows $\Delta i = 0$. The dark purple line shows the running mean and the dashed, pink lines show the running standard deviation from the mean of the Y-band. The running mean of the G-band and I-band is shown in orange. The dotted blue lines show the boundaries of the forbidden regions (see text).
inclination angle being lower than the intrinsic inclination angle. An example of such a galaxy can be seen in Figure 10.

To account for the effect of bulges, we examine sub-samples of galaxies that are more disk-dominated with $\kappa_{rot} > 0.6$, $\kappa_{rot} > 0.7$, and $\kappa_{rot} > 0.8$ (see Section 2 for the definition of $\kappa_{rot}$). In Figure 11, we compare the observed inclination angle and the intrinsic inclination angle, but with subsamples of galaxies that are more disk-dominated. The running mean deviates less from 0 as $\kappa_{rot}$ increases, indicating that the decrease in $\Delta i$ as the intrinsic inclination increases is partially due to bulges and/or other kinematically hot components of the galaxies. And we observe that for galaxies with $\kappa_{rot} > 0.8$, the mean bias is almost negligible. This suggests that the main cause of the systematic bias is the presence of these kinematically hot structures. Meanwhile, it is important to note that the subsamples show nearly the same trend at the low inclination end as the formal sample, which means that the kinematically hot structures are not the cause of the systematic bias at the low inclination angle end.

The root mean square (RMS) of $\Delta i$ for the whole sample is 14.2 degrees, whereas the RMS of $\Delta i$ for the sample of more disk-dominated galaxies is 11.4 degrees. More disk-dominated galaxies result in better measurements of the inclination angle.
Figure 9: The difference in inclination angle $\Delta i$ between the angle calculated in a way similar to observation and the intrinsic inclination angle as a function of the intrinsic inclination angle. The color represents the average $1 - B/A$ at each spatial location. The color coding is further smoothed to highlight the average trends using the two-dimensional LOESS smoothing via robust locally-weighted regression (Cappellari et al., 2013). We only show data within three times the running sigma, and pad empty regions for better visual impressions. The pink line shows the running mean and the dotted blue lines show the boundaries of the forbidden regions as in Figure 7.
Figure 10: A simulated galaxy from our sample with a high intrinsic inclination angle. The disk is embedded in a prominent bulge and halo. AutoProf calculated the ellipticity over the outermost isophote (around the bulge and halo), which can lead to underestimates of the ellipticity.
Figure 11: The distribution of the difference in inclination angle $i$ between the angle calculated in a way similar to observation and the intrinsic inclination angle as a function of the intrinsic inclination angle. The dotted, black line shows $\Delta i = 0$. The running mean of $\Delta i$ for increasingly disk dominated galaxies selected using different values of $\kappa_{\text{rot}}$ is shown.
5. Conclusions

We test the method of measuring the inclination angle of disk galaxies using synthetic images of disk galaxies in the TNG50 cosmological simulation. We find:

1. The lower bound of the observed axis ratio (assumed to be the thickness of the disk) varies with mass.

2. Various $q_0$ assumptions have generally small impacts on the accuracy of inclination angle measurements except at high inclination angles.

3. Different optical bands do not have a noticeable effect on the differences between the intrinsic inclination angles and the observed ones.

4. At intermediate inclination angles, the observed inclination angles are consistent with the intrinsic inclination angles.

5. At higher inclination angles, the presence of bulges, halos and other kinematically hot components results in underestimates of the ellipticities resulting in the observed inclination angle being lower than the intrinsic inclination angle.

6. At lower inclination angles, the observed inclination angle is higher than the intrinsic inclination angle. This may be due to the inaccurate assumption that disks are circular when viewed face on.

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References


Appendix

A. Modifications to AutoProf

The standard AutoProf pipeline preferentially selects certain ellipticities at high and low values of $q$. This results in the ‘bands’ of $q$ values seen in Figure A1. We modify the AutoProf codes in the initialization and isophote fitting stages to better determine the ellipticities of the galaxies and remove this preferential selection. In the initialization stage, we increase the number of grid ellipticities from 15 to 100 and increase the explored range of parameter space to be $\pm 0.2$ around the optimal grid ellipticity (the original interval is exactly the grid interval) when optimizing the continuous ellipticities. In the isophote fitting stage, we increase the perturbation scale from 0.03 to 0.2. This allows AutoProf to explore a broader parameter space when updating the geometric parameters of the isophotes. The downside is that it takes more time to converge, but the benefit is that it increases the chance to escape some local minimums. We also decrease the regularization scale from 1 to 0.1. This allows the adjacent isophotes to have more decoupled geometric parameters, in case they will all be stuck at preferential values if one or a few of them are.

The other minor tweak is to increase the maximum number of iterations of adjusting each isophote before stopping optimization from 1000 to 3000, setting the minimum to 5. We set the number of iterations with no change to parameter before the optimization procedure stops to 5.

However, all of these tweaks dramatically increase the computing time by a factor of $\sim 5$. We are still working on this to find out which of the above are the most important tweaks and to find an optimal solution to balance numerical accuracy and computing time.

Figure A1: The observed axis ratio $q$ as a function of stellar mass where $q$ is calculated using the standard AutoProf pipeline. The distribution of $q$ is shown on the right.